

# The Teacher's Strides for Assisting Students' Thought in Constructing Mathematics Argumentation

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**Abstract.** Playing a crucial role in motivating students that connecting with makes mathematics argumentation is a teacher's job. Some evidence that shows significant student commitment occurs more often in classrooms that centralized to the student. The teacher gathers with the students mutually share argumentation about how to conceive about mathematics concepts that were usually writing in the story problem. However, the teacher is dominating instructional in the class and also scramble to assist student investigation effectively. This paper provides a scaffold of teacher strides specific to investigating how a student makes his argumentation with the correct language and systematically based on the problems they read; the teacher strides for assisting student thought to scaffold. The analysis of four instructors' performances of junior high school students (ages 12–15) shows a research-based unit on ratio and linear equation. The scaffolding organizes pedagogical strides into four categories, eliciting, responding, facilitating, and extending, and then locates individual steps within each group on a continuum according to their goods for assisting student thought. In this means, the teachers' stride scaffold depicts how many teacher strides can collaborate to preserve an investigation-oriented sphere. We break the context with the teacher's steps that show the scaffolding and stages of student argumentation

**Keywords:** *scaffolding, mathematics argumentation, student thought, teachers pedagogy*

## INTRODUCTION

A teacher's reflection for his classroom experience was to know the strengths and weaknesses of the learning strategies implemented so that the practice becomes appropriate or not. Critical incidents that are the result of reflection can come from four things: class management, student prerequisite knowledge, understanding, resistance and motivation, internal facilitators of students and

families, and school organization problems [1]. This reflection also derived from his argumentation when speaking to the students.

Analysis of a teacher's argumentation on pedagogical problems uses the Toulmin model [2]. The results of the analysis show a change in pedagogical argument from a teacher. Teachers need to provide scaffolding assistance to students by pedagogical steps to build mathematical arguments, which also play an essential role in stimulating critical and creative thinking. This category adopts the TMSSR framework, which is to raise, respond, facilitate, and expand [3]. Within these categories, each divided into low to high levels. Scaffolding can also be in the form of modelling of the desired behaviour, offering explanations, inviting student participation, verifying and clarifying student understandings, and inviting students to contribute clues[4].

Argumentative skills are exploratory and broaden student reasoning. The argumentation process accompanied by the ability to express, explain, and argue about the conclusions of an issue [5]. With the teacher's argumentative ability during learning by facilitating students who have difficulty learning or expressing their opinions about a concept, the teacher's role is crucial to help. Students' thoughts about things such as dealing with a problem sometimes also affect their arguments; this can saw in the backing stage of the Toulmin model. Arguments are essential for many learning tasks, to find out to what extent students can restate the problem and analyze and arrange the results of problem-solving systematically. Based on the Theory of Guidance Script, argument scaffolding uses the diagnosis of students' internal argumentative scripts as well as adaptive external support [6].

Proof of mathematical argumentation uses the Toulmin model, which consists of six parts [7]. The first part is Data, which is a known fact and used to prove, Second, is a Claim, which is a statement that is argued or determined. Third, Warrant, general statement, or hypothesis that logically bridges between Claim and Data. Fourth, Qualifiers, the statement that limits an argument that proposes the conditions under which the case is correct. Fifth,

Rebuttal, the counter-argument shows the state when general discussions do not apply. Finally, Backing, a statement supporting Warrant. All these parts can be done or only a few pieces so that the composition of the student's mathematical arguments can be known. Furthermore, students can also arrange part by part according to their reasoning.

**METHOD**

This research uses a case study approach [8], which begins by collecting data from designing learning according to the scaffolding of the teacher's pedagogical steps, namely raising, responding, facilitating, and expanding. The teachers involved were grade seventh junior high school teachers. They helped build student arguments, two related concepts, and two linear equality concepts. Problem stories about comparisons and linear equations require students to understand, model, and be able to find solutions. During the task, the subjects interviewed about the mathematics argument process according to the Toulmin model, which contained six parts, namely 1) data; 2) claim; 3) warrants; 4) qualifier; 5) rebuttal, and 6) blocking.

This study uses video recordings from all groups (teachers and students) during student and teacher interviews, as well as observation sheets. We also provide tests on the concept of ratios and linear equations, each of which consists of 2 questions. Video recording and observation analysis were matching the teacher's step table according to the TMSR framework. This teacher steps divided into low and high levels in each category of eliciting, responding, facilitating, and extending as figure 1.

Eliciting Student Reasoning		Responding to Student Reasoning	
Low ← → High		Low ← → High	
Eliciting Answer	Eliciting Ideas	Correcting Student Error	Prompting Error Correction
Eliciting Facts or Procedures	Eliciting Understanding	Re-voicing	Re-Representing
Asking for Clarification	Pressing for Explanation	Encouraging Student Re-voicing	
Figuring Out Student Reasoning		Validating a Correct Answer	
Checking for Understanding			
Facilitating Student Reasoning		Extending Student Reasoning	
Low ← → High		Low ← → High	
Guiding	Cueing	Providing Guidance	Encouraging Evaluation
	Funneling	Encouraging Multiple Solution Strategies	Encouraging Reflection
	Topaze Effect	Building	Pressing for Precision
Providing	Providing Information	Providing Alternative Solution Strategies	Encouraging Reasoning
	Providing Procedural Explanation	Providing Conceptual Explanation	Pressing for Justification
	Providing Summary Explanation		Pressing for Generalization

Figure 1: the teacher frameworks [3]

Table 1. Instrument for Research

Concept	Problems
Ratio	1. The age of a father is three times the age of his child. If the total age of the father and child at that time is 64 years, how old is the child?
	2. Work completed in 5 days with ten workers. If it fails on days 2, 3, and 4, then it takes five more days?
Linear Equation	3. The number of three consecutive even numbers is 108. Determine the numbers!
	4. A farmer has a rectangular piece of land. The width of the property is six meters shorter than the length. If the ground around the farmer is sixty meters, then determine the land area

**RESULT & DISCUSSION**

The following are the results obtained from each group of teachers and students. The first group uses the instrument one concept ratio problem, the second group uses problem two concept ratios, the third group uses question one concept linear equation, and the fourth group uses problem two concept linear equations.

Table 2. Teacher strides and student's argumentation of ratio concepts (1)

Parts	S1	T1
Data	The age of a father is three times the age of his child.	Eliciting facts (eliciting low) and validating truth (responding low)
Claim	The age of the child is 16 years	Guide claims (facilitating high)
Warrant	- Creating a mathematical model by assuming two variables, father = x, child = y, so x = 3y, and x + y = 64 (modelling) - Equating two known equations (definition of similarity) - Child's age y = 16 (the sum of variables)	Providing guidance in modeling (facilitating high); Encourage thinking and justification (extending high)
Qualifiers	No statement	There are no steps
Rebuttal	No statement	There are no steps
Backing	- Age ratio states the quotient between age variables	Encourage evaluation (extending low)

The results of the first group show all the stages of argumentation from the data, claims, Warrant, and Backing done by students and teachers. Students can restate the problem in the form of a

mathematical model with teacher steps such as eliciting and extending—the conclusion of the concept of the ratio stated by students obtained from the teacher's extending step. T1 encourages students to conduct an evaluation of the problem-solving process and the accompanying reasons.

Table 3. Teacher's strides and student's argumentation of ratio concepts (2)

Parts	S2	T2
Data	Work completed in 5 days with ten workers. It fails on days 2, 3, and 4. number of initial workers = 10, start time = 5, work time = 1, stop time = 3	Bring out understanding (eliciting high) and encourage error correction (responding high)
Claim	There are 30 additional workers	Guide in claiming and providing conceptual explanations (facilitating high)
Warrant	- Creating a mathematical model by assuming the total variable worker = x, so $x + 10 = 50$ , (modeling) - Determine the total workers = 40 people so that the additional workers are 30 people	Urgent explanation (eliciting high)
Qualifiers	No statement	There are no steps
Rebuttal	No statement	There are no steps
Backing	- Finding student time for additional workers (time equality) - Looking for additional workers by finding the total difference of workers with initial workers	Urge students to justify and encourage them to reflect (extending low)

Table 4. Teacher's strides and student's argumentation of linear equation concepts (1)

Parts	S3	T3
Data	There are three consecutive even numbers which number 108	Come up with an idea (eliciting high)
Claim	The even numbers are 34, 36, and 38	Urgent justification (extending high)
Warrant	- Determine the difference between even numbers (even	Restate (responding high); guide to

number patterns) build a model (facilitating high)  
- Make a model of the three numbers, i.e.  $a + (a + 2) + (a + 4) = 108$  (sum of variables)

Qualifiers	No statement	There are no steps
Rebuttal	If it is not an even number, then the pattern cannot be determined	Bring out the understanding (eliciting high)
Backing	A linear equation is an equation with a cubed variable.	Urgent generalization (extending high)

In the second group, it is almost the same as the first group, where there are two stages of argumentation that are not carried out by students, namely qualifiers and Rebuttal. The teacher also does not show steps to help students think and state both stages. Nevertheless, what is different is, the teacher urges students to explain the reasons that accompany the warrant stage and asks students to explain based on the concepts learned

In this third group, there is only one stage of qualifiers that also passed the teacher does not take steps to help that seen from the absence of interaction between the two. The student shows an increase in rebuttal stages, where he can think reversible, which mentions the negation of numbers and patterns that could not determine. The teacher's step, in this case, is to ask students to mention other ways if the problem does not mention the type of the number so that students' understanding of the application arises.

Table 5. Teacher strikes and student's argumentation of linear equation concepts (2)

Parts	S4	T4
Data	The flat rectangular area with length x and width x - 6. Circumference of rectangle 60 meters.	Check understanding and clarification (eliciting low), validate the correct answer (responding low)
Claim	The size of the land is in form of rectangular = $216 \text{ meter}^2$	Provide alternative solution strategies (facilitating high)
Warrant	- Determine the mathematical model of $K = 2x + 2(x-6)$ and $L = x(x-6)$ (the formula for the	Guiding to build models (facilitating high), encouraging to think

	circumference and area of a rectangle)	applications (extending high)
	- Determine the result of $x$ with a substitution (addition of the same variable)	
	- The area of the rectangle determined from the value of $x$	
Qualifiers	The length and width determined from the Data	Encourage a variety of solution strategies (facilitating high)
Rebuttal	No statement	There are no steps
Backing	The area of a rectangle calculated from length $x$ times width $x-6$	Encourages reflection (extending high)

In contrast to the third group, in the fourth argumentation stage, namely qualifiers, the teacher's steps that encourage the search for other solutions, students can state the acquisition of length and width measurements even though the problem not mention. Students tend to be able to state data by identifying known elements, and the teacher immediately confirms the statement.

Each teacher has a difference in scaffolding pedagogical steps when interacting with students who face problems with comparisons and linear equations. The first teacher (T1) did the scaffolding with the step of bringing up the facts when students asked about the first sentence of the problem. Next, students write the first sentence of the argument, then T1 validates the truth of the data statement. T1 assistance when students construct the argumentation of claims and warrant stages is to guide how to model mathematically. Step T1 based on a case that generally describes students' thinking in terms of achieving the correct answer, such as about facts in the division of fractions [9]

The second teacher (T2) starts the interaction by giving rise to an understanding when students ask about writing data. After students write down what is known and what asked, the teacher encourages the correction of the error symbol. In writing a claim in an argument, the teacher guides with a conceptual explanation. This T2 step appears to: identify various forms of mathematical thinking and understanding, such as formal versus informal, procedural versus conceptual definitions, understanding versus memorizing [10].



Figure 2. Teacher's Strides

The third teacher (T3) tries to come up with ideas for students by asking students to read the first sentence of the problem. When students make a claim, the teacher urges them to prove. In the process of finding a solution, the teacher restates the sentence problem with a series of numbers and guides students to build a mathematical model of the even number sequence.

The fourth teacher (T4) checks students' understanding of comparisons by asking students to mention examples. Next, he validates the students' answers correctly and corrects some of the wrong words in the data statement. Claims that, according to him, are not quite right, the teacher provides alternative solution strategies so students can correct them. In writing the completion process, the teacher guides to build a model of equality of the sentence problem, which is the circumference of the building. Steps T3 and T4 in helping students think they are synergistic scaffolding, based on the assumption that each scaffolding will increase the effectiveness of other scaffolding and produce significant interaction effects, namely eliciting, facilitating, responding, returning to facilitating and extending [11].

## CONCLUSION

Based on the results and discussion related to many previous studies, the teacher's steps to help students' thinking in building mathematical arguments divided into low and high levels. The low category is obtained from the teacher's steps in eliciting answers, and responding to correct answers, and encouraging evaluation. Whereas in the high category, the teacher steps in to help students think like eliciting ideas, guide in modelling conceptually, restate, and encourage reflection.

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